Reconstructing f(R) model from Holographic DE: Using the observational evidence

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Abstract

We investigate the corresponding relation between $f(R)$ gravity and an interacting holographic dark energy. By obtaining conditions needed for some observational evidence such as, positive acceleration expansion of universe, crossing the phantom divide line and validity of thermodynamics second law in an interacting HDE model and corresponding it with $f(R)$ mode of gravity we find a viable $f(R)$ model which can explain the present universe. We also obtain the explicit evolutionary forms of the corresponding scalar field, potential and scale factor of universe.

Keywords: Holographic Dark energy; Event horizon; $f(R)$ Gravity.

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1 Introductions

Observational data[1, 2, 3, 4, 5], indicates that the current expansion of universe is accelerating. Several attempts have been made to justified the current accelerated expansion of the universe [6, 7, 8, 9, 11]. One is the presentation of an unknown energy form which is called dark energy. An alternative approach is the modification of the gravitational theory e.g. $f(R)$ gravity in which $f(R)$ is an arbitrary function of the scalar curvature R [6, 13, 15]. Recent various observational data imply that the density of matter (ordinary matter + dark matter+ radiation), $\Omega_m = 0.27$ and the density of dark energy, $\Omega_\Lambda = 0.73$ have capable value today (coincidence problem), beside based on recent data, the equation of state parameter may evolve from $\omega > -1$ (non-phantom phase) in the past to $\omega < -1$ (phantom one) at the present epoch. One way to explain these data, is to consider dynamical dark energy with proper interaction with matter[16].

In the quantum field theory ρ_Λ is regarded as zero-point energy density and defined based on L, the size of the current universe, (dubbed the holographic dark energy) as follow

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \tag{1}$$

where c^2 is a numerical constant of order unity and $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass where G is the Newtonian gravitational constant. Different choices may be adopted for the infrared cutoff of the universe, e.g., Hubble horizon, particle horizon, future event horizon [17]. In a non interaction model, if we take the particle horizon as the infrared cutoff, the accelerated expansion of universe cannot be explained[18], and if the Hubble horizon does chooses as the cutoff, then an appropriate equation of state parameter for dark matter cannot be derived [19]. By taking, the future event horizon as the cutoff, the present expansion of universe may be explained but the coincidence problem still remains unsolved. This problem may be alleviated by considering suitable interaction between dark matter and holigraphic dark energy.

In this paper, we consider a flat friedmann-Robertson- Walker universe and assume that the universe is composed of two interacting perfect fluids, the holographic dark energy and the matter. We assume the infrared cutoff to be a combination of the future and particle event horizon. After some general debate about the properties of the model, we discuss the required conditions needed to cross the phantom divide line in the $f(R)$ model. We show that this crossing imposes some relations between the parameters of the model.

In this paper we well review $f(R)$ model of gravity and make a correspondence between $f(R)$ model and an interacting holographic dark energy model. By investigating the conditions which are needed for describing the present universe, we can obtain a viable $f(R)$ model of gravity.

2 Description and general properties of the model

The equation of motion for the $f(R)$ model is

$$R_{\mu\nu}f' - \frac{1}{2}fg_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu) f' = 8\pi G T_{\mu\nu}, \tag{2}$$

where a prime represents the derivative with respect to the curvature scalar R and $\Box$ is the covariant D'Alembert operator ($\Box \equiv \nabla_\alpha\nabla^\alpha$). We will assume dark energy and cold dark matter perfect fluids with stress-energy tensor given by

$$T_{\mu\nu} = -g_{\mu\nu}p + (\rho + p) u_\mu u_\nu, \tag{3}$$

where ρ and p are the energy density and pressure of the fluid and $u^\mu = (1, 0, 0, 0)$ is its normalized four-velocity in co-moving coordinates. The dark energy component has pressure p_d and energy density ρ_d and the cold dark matter component has zero pressure and energy density ρ_m. The stress-energy tensor is covariantly conserved.

The trace of equation (2) gives an equation of motion for the new scalar degree of freedom (compared to Einsteinian general relativity), [20, 21],

$$3\Box f' = 8\pi G T + 2f - Rf', \tag{4}$$

where T is the trace of the stress-energy tensor. It is helpful to redefine the scalar degree of freedom through

$$\phi = f' - 1. \tag{5}$$

Then Eq. (4) can be reexpressed as an equation of motion for a canonical dimensionless scalar field ϕ with a force term $\mathcal{F}$ and potential V,

$$\Box\phi \;=\; V'(\phi) - \mathcal{F}, \tag{6}$$
$$3V'(\phi) \;=\; 2f - Rf'l7, \tag{7}$$

where the force term that drives the scalar field ϕ is proportional to the trace of the stress-energy tensor, $\mathcal{F} = -8\pi G T/3$.

Now we consider a homogeneous and spatially-flat spacetime with FLRW line element

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{8}$$

where $a(t)$ is the scale factor. The tt component of the gravitational equations (2), for the metric (8), can be simplified to

$$H^2 + H\frac{d}{dt}\left(\ln f'\right) - \frac{1}{6}\left(\frac{f - Rf'}{f'}\right) = \frac{8\pi G}{3f'}\rho_m. \tag{9}$$

The Friedmann equation, (6), can be written in a somewhat more conventional form as

$$H^2 = \frac{8\pi G}{3\left(1 + \phi\right)}\left(\rho_m + \rho_d\right), \tag{10}$$

where we assume that the new scalar degree of freedom behaves like dark energy with dark energy density

$$\rho_d = -\frac{3\left(1 + \phi\right)}{8\pi G}\left[H\frac{d(\ln f')}{dt} - \frac{1}{6}\left(\frac{f - Rf'}{f'}\right)\right]. \tag{11}$$

Also, one can write the Friedmann equation as

$$\Omega_m + \Omega_d = 1, \tag{12}$$

where the density parameters $\Omega_m = \rho_m/\rho_c$, $\Omega_d = \rho_d/\rho_c$, and the critical energy density is

$$l13\rho_c = \frac{3H^2(1 + \phi)}{8\pi G}. \tag{13}$$

From the Friedmann equation (10) and conservation of stress-energy tensor, we have

$$\dot{H} = -\frac{4\pi G\left(\rho_t + p_d\right)}{1 + \phi} - \frac{\dot{\phi}H}{2(1 + \phi)}, \tag{14}$$

where $\rho_t = \rho_m + \rho_d$. The vanishing of the covariant divergence of the stress-energy tensor for the whole system gives the conservation equation in the metric (8),

$$\dot{\rho}_t + 3H\left(\rho_t + p_d\right) = 0. \tag{15}$$

But, because of interactions between the two components, each individual component is not necessarily conserved. So, one can write

$$\dot{\rho}_d + 3H\left(\rho_d + p_d\right) = -Q, \tag{16}$$

$$\dot{\rho}_m + 3H\rho_m = Q.l17 \tag{17}$$

We consider different forms of Q below. A number of different models have been proposed for dark energy. Here we want to investigate the holographic dark energy model and see if it can be related to the $f(R)$ model. Holographic dark energy is described in terms of an infrared cut-off length, L, and the energy density is defined as

$$l18\rho_d = \frac{3c^2 M_p^2}{L^2}.$$

(18)

where c^2 is a constant of order unity and M_p is the Plank mass. This is motivated by quantum theory of gravity considerations, in particular the holographic principle [18, 19, 25]. It was shown in [26] that in quantum field theory the UV cutoff Λ should be related to the IR cutoff L due to a limit set by forming a black hole with Schwarzschild radius L. If $\rho_d = \Lambda^4$ is the vacuum energy density of the UV cut-off scale, the total energy in volume L^3 should not exceed the mass of the system-size black hole. This means that $L^3\rho_d \leq M_p^2 L$. So for the largest cut-off L, one can define the holographic dark energy as (??). From Eqs. (??) and (??), the density parameter of holographic dark energy can written as

$$\Omega_d = \frac{c^2}{(1 + \phi) H^2 L^2}.$$

(19)

The IR cut-off, L, is presumably determined by the available length scale. To retain generality, we assume that it is a linear combination of the particle horizon, R_p, and the future event horizon, R_f, i.e., we choose L to be

$$L = \alpha R_f + \beta R_p,$$

(20)

where

$$R_f = a(t) \int_t^\infty \frac{dt}{a(t)}, \qquad R_p = a(t) \int_{t_{\min}}^t \frac{dt}{a(t)},$$

(21)

here $t_{\min}$ is the time when the particle was created, and $0 \leqslant \alpha, \beta \leqslant 1$ and $\alpha + \beta = 1$. For $\alpha = 1$, $\beta = 0$ we get $L = R_f$ while $\alpha = 0$, $\beta = 1$ gives $L = R_p$.

Taking the time derivative of (??), and using (16), one can obtain the equation of state parameter $\omega_d = p_d/\rho_d$,

$$l22\omega_d = -\frac{1}{3}\left[1 + \frac{Q}{H\rho_d} - \frac{2(\beta - \alpha)}{HL}\right].$$

(22)

5

To progress, we have to specify the interaction term Q. A generic form of Q is not available. Three forms which are often discussed in the literature are $Q = 3b^2 H \rho_d, 3b^2 H \rho_m, 3b^2 H \rho_t$, where b^2 is a constant which has to be positive, because following the second law of thermodynamic, energy transfer can only be from dark energy to cold dark mater. These three forms of interaction give almost the same result, so for definiteness, we choose

$$l23 \quad Q = 3b^2 H \rho_d. \tag{23}$$

Using (19), (**??**) and (**??**), we find

$$l24 \quad \omega_d = -\frac{1}{3}\left[1 + 3b^2 - \frac{2(\beta - \alpha)}{c}\sqrt{(1 + \phi)\Omega_d}\right]. \tag{24}$$

As mentioned in the Introduction, observational data indicate that the current cosmological expansion is accelerating. In the fluid model this accelerated expansion requires $\omega_d < -1/3$. This constraint results in

$$l25 \quad \frac{2(\beta - \alpha)}{c}\sqrt{(1 + \phi)\Omega_d} < 3b^2. \tag{25}$$

Defining a positive constant $0 < k_0 < 1$, we can rewrite (**??**) as

$$l26 \quad \frac{2(\beta - \alpha)}{c}\sqrt{(1 + \phi)\Omega_d} = 3k_0 b^2. \tag{26}$$

The second law of thermodynamics requires that the entropy S increasing with time then, $\dot{S} > 0$. We assume S is the entropy attribute to the surface area $A = 4\pi L^2$, where L is the infrared cut-off length appearing in (**??**). Also, making use of Nother charge method, one can obtain the entropy in the $f(R)$ model of gravity for a horizon with surface $A = 4\pi L^2$ as [23]

$$S = \frac{Af'(R)}{4} = \pi L^2(1 + \phi), \tag{27}$$

Then, considering the thermodynamics second law, the time derivative of entropy, S, should be

$$l28 \quad \frac{\dot{S}}{\pi L^2} = \left[2H + \frac{\dot{\phi}}{1 + \phi} + \frac{2(\beta - \alpha)}{L}\right] \geq 0. \tag{28}$$

We set (**??**) as

$$l29 \quad \left[2H + \frac{\dot{\phi}}{1 + \phi} + \frac{2(\beta - \alpha)}{L}\right] = s_0, \tag{29}$$

6

where $0 \leq s_0$. $s_0 = 0$ is when the accelerating expansion of the horizon of universe be adiabatic. Here we assume $s_0 > 0$ and then

$$l30 \frac{2(\beta - \alpha)}{HL} = s_0 - \frac{\dot{\phi}}{H(1 + \phi)} - 2, \tag{30}$$

By making use of (**??**) and (**??**), we have

$$l31 \frac{\dot{\phi}}{H(1 + \phi)} = s_0 - 3k_0 b^2 - 2 = \theta_0, \tag{31}$$

On the other hand, based on recent data, the dark energy component seems to have an equation of state parameter $\omega_d < -1$ at the present epoch, while $\omega_d > -1$ in the past [24]. Therefore, we expect the equation of state parameter cross the phantom divide line, then when $\omega = -1$, the crossing is allowed. So by implying the phantom crossing line constraint on ω, (**??**), we have

$$l32 \frac{2(\beta - \alpha)}{HL} = 3b^2 - 2. \tag{32}$$

From (**??**) and (**??**) we have

$$l330 < k_0 = 1 - \frac{2}{3b^2} < 1, \tag{33}$$

this show that $0 < 2/3b^2 < 1$. This means that one of the constant can be omit. Moreover, to cross $\omega_d = -1$, $\dot{\omega}_d$ must be negative at the transition time, $\omega_d = -1$. So by using (**??**) and time derivative of (**??**) we have

$$l34 \dot{\omega} = -\left(\frac{\dot{H}}{H^2} + \frac{3}{2}b^2 \right) \left(b^2 - \frac{2}{3} \right) < 0. \tag{34}$$

From (**??**), we have $b^2 > 2/3$, so that the relation (**??**) is satisfied when

$$l35 \frac{\dot{H}}{H^2} > -\frac{3}{2}b^2 \tag{35}$$

So solving (**??**), gives

$$l36 H = \frac{h_0}{1 + h_0 \gamma t}, \tag{36}$$

where $\gamma = 3\xi_0 b^2/2$, ξ_0 is an arbitrary constant which satisfy $0 < \xi_0 < 1$ condition. We assume $H_0 t_0 \sim 1$ (H_0 and t_0 are Hubble parameter in the present time respectively) then $h_0 = H_0/(1 - \gamma)$. By making use of (**??**)

7

and (**??**) we can easily find the scale factor of universe and the scalar field ϕ as

$$l37 \quad a(t) = a_0(1 + h_0\gamma t)^{\frac{1}{\gamma}}, \tag{37}$$

$$\phi(t) = -1 + \phi_0(1 + h_0\gamma t)^{\frac{\theta_0}{\gamma}}, l38 \tag{38}$$

where $a_0 = (1 - \gamma)^{1/\gamma}$ (we assume that the scale factor in the present time is equal to 1, $a(t_0) = 1$) and ϕ_0 are the integration constants. It is clearly seen that, from (**??**), at the early time $a(0) \neq 0$ and then we have bouncing in the beginning of the universe. It is well known that the Rcci scalar in flat FLRW is as

$$l39 \quad R = 6\left[\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2\right], \tag{39}$$

then

$$l40 \quad R = \frac{R_0}{(1 + h_0\gamma t)^2}, \tag{40}$$

where $R_0 = 6h_0^2(1 - 2\gamma)$. So that using (5), (**??**) and (**??**), one can obtain a viable $f(R)$, which allow the crossing from $\omega = -1$, as

$$l41 \quad f(R) = f_0 + CR^{1-\epsilon}, \tag{41}$$

here $C_0 = \phi_0 R_0^{\theta/2\gamma}/\epsilon$, $\epsilon = \theta/2\gamma$ and f_0 is the constant of integration which can be as well as cosmological constant, Λ. By making use of chameleon mechanism this kind of $f(R)$ model has been studied in [27] and they show that this form of $f(R)$ model, is viable and satisfy the observational constraints solar system. Also using (**??**) and (**??**) one can rewrite (**??**) as

$$l42 \quad V(\phi) = V_0 + V_1\phi + \frac{V_2}{(1 + \phi)^{2\gamma\theta}}, \tag{42}$$

where V_0 is a constant of integration, $V_1 = 3f_0/2$ and

$$V_2 = \frac{\epsilon(1 + \epsilon)\phi_0^{\frac{1}{\epsilon}}R_0}{(1 - \epsilon)(2\epsilon - 1)}$$

3   Conclusion

The HDE model is an attempt for probing the nature of DE within the framework of quantum gravity [28]. In this work we used of HDE model which is in interaction with DM in the flat FLRW universe. We established

8

a correspondence between the interacting HDE model with the $f(R)$ model of gravity in the flat FLRW universe. These correspondences are important to understand how different models which have been candidated for explanting the present universe, are mutually related to each other. However, by taking account an infrared cutoff as a combination of particle and future event horizons and using the HDE energy density, we obtained the EoS parameter for the interacting HDE. Using equations derived for equation of state parameter of holographic dark energy and its time derivative, condition required for crossing the phantom divide line was derived. Also the condition of validity of thermodynamics second law for the infrared cutoff was obtained. Thus we studied the evolving behavior of the interacting HDE and implying some observational evidence such as, positive acceleration expansion of the universe ($\omega < -1/3$ and $q < 0$), crossing the phantom divide line ($\omega < -1$) and validity os second law of thermodynamics for an interacting model of HDE, we reconstructed the $f(R)$ model which describe accelerated expansion of the universe. We also obtained the explicit evolutionary forms of the corresponding scalar fields, potential and scale factor of universe.

4   Aknowledgement

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